Digital Television Techniques

Exercise 3, FEC/COFDM, 26-April-2012

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Deadline is 7 May 2012.

1. The *Pseudo Random Binary Sequence* in DVB-T is generated using 15 shift registers and the polynomial generating sequence

   $$1 + X^{14} + X^{15}$$

   and the shift registers initially are loaded with the values $\{0,0,0,0,0,0,0,1,0,1,0,1,0,0,1\}$.

   Generate the PRBS signal for the sequence

   $$\{0,0,0,0,1,1,1,0,1,0,1\}$$

2. A digital system uses a convolutional encoder where the generator polynomials are given by $G_1 = 6_{Oct}$ and $G_2 = 3_{Oct}$. Sketch the encoder. Sketch the trellis diagram for the code.

3. A black and white TV screen has approximately $3 \cdot 10^5$ image elements (pixels) which can have one of ten distinct intensity levels with the same probability. Suppose that the transmission rate is 25 pictures per second and that the signal-to-noise ratio is 30 dB. Find the necessary bandwidth of the channel. Assume the Shannon's theorem $C = B \log_2(1 + S/N)$.

4. The binary signal {01000100011010} is sent using QPSK (Quadrature Phase Shift Keying), using constellations (bits) 0 (00) , $\pi/2$ (01), π (10) and $3\pi/2$ (11). The symbol length is $2T_c$ where $T_c = 1/f_c$, amplitude $A_c = 1$. Draw the modulated signal $x_c(t)$ for $t \in [0, 16T_c]$.

5. A multi-path channel shows a maximum time delay of $\tau_{max} = 0,8\mu s$. How long should the Guard interval (Cyclic Prefix) of an OFDM system with $N = 64$ sub-carriers and a carrier spacing of $\Delta f = 312,5$kHz be dimensioned for the transmission over the channel? Note, select among guard intervals of length 1/4, 1/8, 1/16, 1/32 etc.

6. A OFDM system has the following parameters: Mode 8k (6817 used carriers), Code rate 2/3, Reed-Solomon (204,188), bandwidth 7 MHz, 1/16 Guard interval, 64QAM constellation, 105 pilot carriers. What is the a) symbol length T_s, b) frequency spacing Δf and c) bit rate B of the transport stream? e) What is the maximum extra distance a transmission echo may travel in order not to cause interference?

   Repeat the calculations for a 2k (1705 used carriers) system with the same parameters.

7. The Discrete Fourier and Inverse Discrete Fourier transforms are important parts of an OFDM transmission system. The Inverse Discrete Fourier transform is defined as

   $$x\left[kT\right] = \frac{1}{N} \sum_{n=0}^{N-1} X\left[\frac{n}{NT}\right] e^{j2\pi nk/N}$$

   where T is the sampling period, N is the size of the IDFT window x are the time samples and X are frequency samples, $k \in [0, K]$, normally $K = N$. Hence, for calculating an IFT we need N^2 complex multiplications and $N \cdot (N-1)$ complex additions, or a complexity $O(N^2)$ (actually totally $4N^2$ real multiplications and $4N^2 - 2N$ real additions).

   The Fast Fourier Transform (IFFT) reduces the computing complexity to $O(N \log_2 N)$. For the systems given in task 2, calculate the computing power in MFLOPS (Mega Floating Point Operations Per Second) for both standard IDFT an IFFT. *Compare 2,8 GHz Core i7 about 2000 MFLOPS (Linpack).*

8. **BONUS** In case of OFDM, a higher bandwidth efficiency than with FDM is reached by overlapping the sub-spectra, which is possible due to the orthogonality of the signal constellations. Different oscillator frequencies of the transmitter and receiver as well as the time selectivity of the channel (Doppler shift) result in a frequency offset $\nu = (n + m)\Delta f$ (with $\Delta f = 1/T_s$ as the carrier spacing). Illustrate that in this case the orthogonality of the sub-carriers is violated at the receiver.