Digital Television Techniques

Exercise 4, COFDM, 22-April-2010

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Deadline is 6 May 2010.

1. A multi-path channel shows a maximum time delay of $\tau_{max} = 0,8\mu s$. How long should the Guard interval (Cyclic Prefix) of an OFDM system with $N = 64$ sub-carriers and a carrier spacing of $\Delta f = 312,5$kHz be dimensioned for the transmission over the channel? Note, select among guard intervals of length 1/4, 1/8, 1/16, 1/32 etc.

2. A OFDM system has the following parameters: Mode 8k (6817 used carriers), Code rate 2/3, Reed-Solomon (204,188), bandwidth 7 MHz, 1/16 Guard interval, 64QAM constellation, 105 pilot carriers. What is the a) symbol length T_s, b) frequency spacing Δf and c) bit rate B of the transport stream? e) What is the maximum extra distance a transmission echo may travel in order not to cause interference?

   Repeat the calculations for a 2k (1705 used carriers) system with the same parameters.

3. The Discrete Fourier and Inverse Discrete Fourier transforms are important parts of an OFDM transmission system. The Inverse Discrete Fourier transform is defined as

$$x\left[kT\right] = \frac{1}{N}\sum_{n=0}^{N-1} X\left[\frac{n}{NT}\right] e^{j2\pi nk/N}$$

   where T is the sampling period, N is the size of the IDFT window x are the time samples and X are frequency samples, $k \in [0, K]$, normally $K = N$. Hence, for calculating an IFT we need N^2 complex multiplications and $N \cdot (N-1)$ complex additions, or a complexity $O(N^2)$ (actually totally $4N^2$ real multiplications and $4N^2 - 2N$ real additions).

   The Fast Fourier Transform (IFFT) reduces the computing complexity to $O(N \log_2 N)$. For the systems given in task 2, calculate the computing power in MFLOPS (Mega Floating Point Operations Per Second) for both standard IDFT an IFFT. *Compare 1 GHz Athlon about 700 MFLOPS.*

4. **BONUS** In case of OFDM, a higher bandwidth efficiency than with FDM is reached by overlapping the sub-spectra, which is possible due to the orthogonality of the signal constellations. Different oscillator frequencies of the transmitter and receiver as well as the time selectivity of the channel (Doppler shift) result in a frequency offset $\nu = (n + m)\Delta f$ (with $\Delta f = 1/T_s$ as the carrier spacing). Illustrate that in this case the orthogonality of the sub-carriers is violated at the receiver.